

# **ILIAD TESTING; AND A KALMAN FILTER FOR 3-D POSE ESTIMATION**

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## **ABSTRACT**

This report presents the results of a two-part project. The first part presents results of performance assessment tests on an Internet Library Information Assembly Data Base (ILIAD). It was found that ILIAD performed best when queries were short (one-to-three keywords), and were made up of rare, unambiguous words. In such cases as many as 64% of the typically 25 returned documents were found to be relevant. It was also found that a query format that was not so rigid with respect to spelling errors and punctuation marks would be more user-friendly.

The second part of the report shows the design of a Kalman Filter for estimating motion parameters of a three dimensional object from sequences of noisy data derived from two-dimensional pictures. Given six measured deviation values representing X, Y, Z, pitch, yaw, and roll, twelve parameters were estimated comprising the six deviations and their time rate of change. Values for the state transition matrix, the observation matrix, the system noise covariance matrix, and the observation noise covariance matrix were determined. A simple way of initializing the error covariance matrix was pointed out.

## INTRODUCTION

A two-part project was undertaken. For the first part, some tests were performed on ILIAD( Internet Library Information Assembly Data Base)[1] in order to assess its performance, user-friendliness, and to develop some insight into the search technique utilized by ILIAD. For the second project, A Kalman Filter was designed for estimating a rigid body's motion parameters from noisy images. The ILIAD tests will be describe first, followed by the Kalman Filter Design.

## THE ILIAD TESTS

### Introduction.

ILIAD, which is an Internet-based information search and retrieval system with a built-in intelligent agent, was designed and implemented, at the NASA Johnson Space Center, by the Client Server Branch of the Information Systems Directorate. It was designed as an intelligent data base primarily to serve K-12 teachers. In operation, a teacher sends a "query", made up of key words, to ILIAD, via electronic mail(e-mail). ILIAD uses the key words to search the Internet for documents whose contents deal with the subject matter represented by the key words. The entire contents of these documents are sent, by e-mail, to the person who sent the query.

It must be emphasized, that apart from the tests described here, the designers of ILIAD performed their own series of tests. What is reported here is, for the most part, the result of the author's tests. Although these tests are not comprehensive, their value lies in the fact, that they provide a sense of how well ILIAD is working---more than thirty queries were generated and sent by the author---, and help point to some of the issues that must be addressed in any future systematic testing of, and improvements upon, ILIAD. The queries employed dealt with subject matter that the author was interested in.

### Testing Procedure:

The testing process comprised the following:

1. Develop a series of queries spanning a variety of subject areas.
  2. Send queries to ILIAD.
  3. Observe simplicity/complexity of query submission and response.
  4. Examine ILIAD responses, noting:
    - a) Response Time.
    - b) Relevance of Returned Documents.
  5. Select a few representative queries and the corresponding returned documents.
  6. Examine these documents in detail with respect to:
    - a) Ratio of relevant versus non-relevant documents
    - b) Choice of words or terminology and the relevance of returned documents.
- b) ILIAD's relevance criteria versus query submitter's relevance criteria:  
--- "Key words search" or "Key phrases" search?

## RESULTS AND DISCUSSION.

### Query Format, Simplicity And User Friendliness.

The author found the query format and ILIAD acknowledgements to be quite simple and user-friendly. The user only needs to type in the following:

\*TeacherEmail: the user's actual email is typed here.

\*VolumeLevel: type in here, low, medium, or high, to indicate amount of materials desired.

\*?Q1: actual query key words are typed in here.

Problems arise only when the user is careless in typing in the required items in the exact format specified. This would typically be in the form of typographical errors, omissions, and incorrect cases (upper case or lower case) for characters. Although, it has been pointed out that most email systems have spell checkers, rejection of queries by ILIAD due to incorrect formats could be a source of frustration for teachers, many of whom may be in a hurry to squeeze in some information retrieval activity among their daily busy schedule.

Upon the author's recommendations, the ILIAD designers have implemented changes that make ILIAD no longer rigidly sensitive to spelling errors and innocuous characters like spaces and commas.

### ILIAD Response Time And Relevance Of Returned Documents.

When a query was sent, ILIAD first acknowledged receipt of the query within a minute. The documents themselves were received within one to two hours.

More than thirty queries were submitted. These dealt with a variety of subject areas including technology, science, government, history, and ancient architecture. Some representative queries were: "parallel processing", "genetic algorithms", "intelligent robots", "fuzzy logic", "microprocessors microcontrollers", "wavelets", "European Economic Community", and "Egyptian Pyramids". Most of the documents returned in response to these queries were found to be very relevant. However, documents returned in response to queries like: "Microsoft Corporation", "wavelets communication", "volcano eruptions Africa", were not particularly relevant. More on that later, under the section on "Key words search" or "Key phrases search".

### Detailed Results Of Some Queries.

Shown below is a tabulation of the details of six queries.

Query #	Query	# Of Docs.	# Docs. Relv.	% Docs. Relv.
1	parallel processing	25	10	40%
2	intelligent robots	25	12	48%
3.	wavelets	25	16	64%
4.	Airbus Consortium	25	0	0%

5.	wavelet communications	25	< 5	<20%
6.	microprocessors	25	>15	>60%

**Key Word Choice And Relevance Of Returned Documents.**

The data shown above, together with other observed results, would seem to indicate that the ideal query would be a one word query where that single word does not find usage in many different applications and contexts. For example, the query “wavelet” produces a high percentage of relevant documents(64%), whereas the query “Airbus Consortium” produces a low percentage of relevant documents( 0%)! Although the term, “Airbus” generally refers to the model name of the airplane manufactured by the European Airbus Industries Inc., the term “Consortium” has such widespread usage, that ILIAD found lots of documents describing consortiums that had nothing to do with the Airbus Industries of Europe.

**“Key Words” Or “Key Phrases” Search?**

ILIAD uses the WAIS(Wide Area Information Server)[2] package developed at Thinking Machines Corporation, Cambridge, Massachusetts. In his paper entitled, “Massively Parallel Information Retriever for Wide Area Information Servers”, Craig Stanfill of Thinking Machines Corporation refers to the way in which WAIS treats the query thus: [3]

“ Queries consist of short natural language phrases, such as ‘Corazon Aquino and the Philippine Election’. Each phrase is broken into primitive components such as ‘Corazon Aquino,’ and ‘Philippine Election,’ and each component is assigned a numerical weight with rare(i.e. more specific)terms assigned higher value. The documents are then ranked from highest to lowest, and the best matches presented to the user.”

The author has not been able to assess how efficiently WAIS does this weight assignment in favor of ‘rare terms’, but in the case of the query “wavelet communication”-communication, meaning information transmission in an engineering context--, the term ‘communication’ must not have been assigned a small enough weighting, because too many documents containing only the word ‘communication’(within a non-engineering context), and without the word ‘wavelet’, were returned by ILIAD. A higher percentage of relevant documents would result if WAIS did indeed search for the occurrences of the whole phrase representing the key words. Alternatively, a more stringent assignment of weighting in favor of rare terms, so that the word ‘communication’ would be recognized as an everyday word , and therefore should be assigned a very low weighting wherever it occurs alone.

**KALMAN FILTER DESIGN**

**Introduction.**

The second project involved the design of a Kalman filter for estimating motion parameters of a three dimensional body from a sequence of two dimensional images. Geometrical techniques had been used by Jodi Seaborn and Robert Goode of ER4 to obtain measured values of the object’s center coordinates, X,Y,Z, as well as the pitch, roll, and

yaw, as determined from at least three image corresponding points on the object and on the two-dimensional picture.[4] The idea is to use the Kalman filter to estimate these parameters in addition to their time rates of change.

**System Modeling, Determining The State Transition Matrix[5,6,].**

The object to be tracked can be modelled by the state equations

$$x(k+1) = \Phi x(k) + u(k) \tag{1}$$

where

$$x(k) = \begin{matrix} X(k) \\ X(k) \\ Y(k) \\ Y(k) \\ Z(k) \\ Z(k) \\ \alpha(k) \\ \alpha(k) \\ \beta(k) \\ \beta(k) \\ \chi(k) \\ \chi(k) \end{matrix}$$

$$= \begin{bmatrix} X - deviation \\ X - rate \\ Y - deviation \\ Y - rate \\ Z - deviation \\ Z - rate \\ pitch - deviation \\ pitch - rate \\ yaw - deviation \\ yaw - rate \\ roll - deviation \\ roll - rate \end{bmatrix}$$

Assume an object being tracked is at coordinates  $X'+X(k)$ ,  $Y'+Y(k)$ ,  $Z'+Z(k)$ ,  $A + \alpha(k)$ ,  $B + \beta(k)$ ,  $X + \chi(k)$ , at time  $k$ , and at coordinates  $X'+X(k+1)$ ,  $Y'+Y(k+1)$ ,  $Z'+Z(k+1)$ ,  $A + \alpha(k+1)$ ,  $B + \beta(k+1)$ ,  $X + \chi(k+1)$ , at time  $k+1$ ,  $T$  seconds later. "T" represents the time spacing between two successive two dimensional still images taken by a camera of the moving object. We are interested in estimating these linear and angular deviations and their rates, which are assumed to be statistically random with zero-mean values.

To a first approximation, if the object is moving at velocities, or rates, given by  $\dot{X}(k)$ ,  $\dot{Y}(k)$ ,  $\dot{Z}(k)$ ,  $\dot{\alpha}(k)$ ,  $\dot{\beta}(k)$ ,  $\dot{\chi}(k)$ , and  $T$  is not too large, then considering, for example, the  $Z$  coordinate, we have

$$Z(k+1) = Z(k) + T\dot{Z}(k) \dots\dots\dots(2)$$

which is an example of a "deviation equation".

Similarly, considering the acceleration  $u(k)$ , we have

$$Tu(k) = \dot{Z}(k+1) - \dot{Z}(k) \dots\dots\dots(3)$$

which is the "acceleration equation". Assuming that  $u(k)$  is a zero-mean, stationary white noise process, the acceleration is, on the average, zero and uncorrelated between intervals,  $E[u(k+1)u(k)] = 0$ , but it has some variance  $E[u^2(k)] = \sigma_u^2$ . Such accelerations could be caused by short term irregularities in external influences on the object. The quantity  $u_3(k) = Tu(k)$  is also a white noise process, and therefore the acceleration equation for the coordinate  $Z$ , can be written as

$$\dot{Z}(k+1) = \dot{Z}(k) + u_3(k) \dots\dots\dots(4)$$

The complete set of range/bearing and acceleration equations for the twelve parameters are:

$$\begin{aligned}
 X(k+1) &= X(k) + T\dot{X}(k) \\
 \dot{X}(k+1) &= \dot{X}(k) + u_1 \\
 Y(k+1) &= Y(k) + T\dot{Y}(k) \\
 \dot{Y}(k+1) &= \dot{Y}(k) + u_2 \\
 Z(k+1) &= Z(k) + T\dot{Z}(k) \\
 \dot{Z}(k+1) &= \dot{Z}(k) + u_3 \\
 \alpha(k+1) &= \alpha(k) + T\dot{\alpha}(k) \\
 \dot{\alpha}(k+1) &= \dot{\alpha}(k) + u_4 \\
 \beta(k+1) &= \beta(k) + T\dot{\beta} \\
 \dot{\beta}(k+1) &= \dot{\beta}(k) + u_5 \\
 \chi(k+1) &= \chi(k) + T\dot{\chi} \\
 \dot{\chi}(k+1) &= \dot{\chi}(k) + u_6
 \end{aligned} \dots\dots\dots(5)$$

$$u(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \\ u_4(k) \\ u_5(k) \\ u_6(k) \end{bmatrix} = \begin{bmatrix} X - \text{acceleration} \\ Y - \text{acceleration} \\ Z - \text{acceleration} \\ \text{pitch} - \text{acceleration} \\ \text{yaw} - \text{acceleration} \\ \text{roll} - \text{acceleration} \end{bmatrix} \dots\dots \text{between time } k \text{ and } k+1$$

The complete state equation for the system is given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ x_5(k+1) \\ x_6(k+1) \\ x_7(k+1) \\ x_8(k+1) \\ x_9(k+1) \\ x_{10}(k+1) \\ x_{11}(k+1) \\ x_{12}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \\ x_7(k) \\ x_8(k) \\ x_9(k) \\ x_{10}(k) \\ x_{11}(k) \\ x_{12}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ u_1(k) \\ 0 \\ u_2(k) \\ 0 \\ u_3(k) \\ 0 \\ u_4(k) \\ 0 \\ u_5(k) \\ 0 \\ u_6(k) \end{bmatrix} \dots\dots(6)$$

The measured data( the six motion parameters) are assumed to be noisy, and are modelled thus:

$$\begin{array}{l}
 y_1(k) = x_1(k) + v_1(k) \\
 y_2(k) = x_3(k) + v_2(k) \\
 y_3(k) = x_5(k) + v_3(k) \\
 y_4(k) = x_7(k) + v_4(k) \\
 y_5(k) = x_9(k) + v_5(k) \\
 y_6(k) = x_{11}(k) + v_6(k)
 \end{array}
 \left\{ \begin{array}{l}
 X - deviation \\
 Y - deviation \\
 Z - deviation \\
 pitch - deviation \\
 yaw - deviation \\
 roll - deviation
 \end{array} \right. \dots\dots\dots(7)$$

Therefore, the data vector can be written as:

$$\begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \\ y_4(k) \\ y_5(k) \\ y_6(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \\ x_7(k) \\ x_8(k) \\ x_9(k) \\ x_{10}(k) \\ x_{11}(k) \\ x_{12}(k) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ v_2(k) \\ v_3(k) \\ v_4(k) \\ v_5(k) \\ v_6(k) \end{bmatrix} \dots\dots(8)$$

In terms of vector formulation, the two vector equations representing the system and measurement models are:

$$\begin{array}{l}
 x(k+1) = \Phi(k)x(k) + u(k) \\
 y(k) = H(k)x(k) + v(k)
 \end{array} \dots\dots\dots(9)$$

$$\Phi = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \text{state transition matrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

is the observation matrix.

The additive noise,  $v(k)$ , is usually assumed to be Gaussian with zero-mean and variances  $\sigma_x^2(k)$ ,  $\sigma_y^2(k)$ ,  $\sigma_z^2(k)$ ,  $\sigma_\alpha^2(k)$ ,  $\sigma_\beta^2(k)$ ,  $\sigma_x^2(k)$ .

The next step is to formulate noise covariance matrix  $Q$  for the system, and  $R$  for the measurement model.

Since we are assuming there is no correlation between noise processes, either in the case of system noise processes, or in the measurement noise processes, the off-diagonal terms of both the observation noise covariance matrix,  $R$ , and the system noise covariance matrix,  $Q$ , are all zero.

For the measurement model, the noise covariance matrix is given by

$$R(k) = E[v(k)v^T(k)] = \begin{bmatrix} \sigma_x^2(k) & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_y^2(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_z^2(k) & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\alpha^2(k) & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\beta^2(k) & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_x^2(k) \end{bmatrix} \dots(10)$$

and the system noise covariance matrix is, for this case, given by

$$Q(k) = E[u(k)u^T(k)] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_2^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_3^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_4^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_5^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_6^2 \end{bmatrix} \dots(11)$$

where

$\sigma_1^2 = E(u_1^2)$ ,  $\sigma_2^2 = E(u_2^2)$ ,  $\sigma_3^2 = E(u_3^2)$ ,  $\sigma_4^2 = E(u_4^2)$ ,  $\sigma_5^2 = E(u_5^2)$ ,  $\sigma_6^2 = E(u_6^2)$ .  
are the variances of T times the linear and angular accelerations respectively.

Specific values must be substituted for those variances in order to define the Kalman filter numerically. One way to proceed is to assume that the probability density function(p.d.f) for each of the six accelerations is uniform and equal to  $p(u)=1/2M$ , between limits +M and -M. The variance then is  $\sigma_u^2 = M^2 / 3$ .

Therefore, the six variances for the system noise covariance matrix are:

$$\begin{aligned} \sigma_1^2 &= \sigma_2^2 = \sigma_3^2 = T^2 \sigma_u^2 \\ \sigma_4^2 &= \sigma_1^2 / Y'^2 \\ \sigma_5^2 &= \sigma_1^2 / X'^2 \\ \sigma_6^2 &= \sigma_1^2 / R^2 \end{aligned} \dots\dots\dots(12)$$

where X', Y' is the average X, Y coordinates, R is the radial distance of the image corresponding points, all taken with respect to the center of the object.

One other covariance matrix must be determined. This is the "error covariance matrix", P. It is the mean-square errors of all the estimates over their actual values. Its vector form is:

$$P(k) = E[e(k)e^T(k)] \dots\dots\dots(13)$$

For twelve signals, we have a 12x12 matrix, thus:

$$P(k) = \begin{bmatrix} E[e_1^2(k)] & E[e_1(k)e_2(k)] & \dots & \dots & \dots & \dots & E[e_1(k)e_{12}(k)] \\ \cdot & E[e_2^2(k)] & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ E[e_{12}(k)e_1(k)] & \cdot & \cdot & \cdot & \cdot & \cdot & E[e_{12}^2(k)] \end{bmatrix} \dots\dots\dots(14)$$

**The Kalman Filter And The Computational Process.**

A form of the Kalman Filter equations suitable for numerical computation is indicated below.

Estimator:

$$\hat{x}(k) = \Phi\hat{x}(k-1)H + K(k)[y(k) - H\Phi\hat{x}(k-1)]$$

Filter - gain:

$$K(k) = P_1(k)H^T [HP_1(k)H^T + R(k)]^{-1} \dots\dots\dots(15)$$

where.....  $P_1(k) = \Phi P(k-1)\Phi^T + Q(k-1)$

Error...covariance..matrix:

$$P(k) = P_1(k) - K(k)H(k)P_1(k).$$

To start Kalman processing we have to initialize the gain matrix K(k). For this purpose, the error covariance matrix P(k) has to be specified in some way. A reasonable initialization can be established using a sequence of two cosecutive measurements, in this case, two consecutive images. This is meaningful in situations where actual initial deviation values are known. This will give six linear and angular deviations at time k=1, and six linear and angular deviations at time k=2. From these twelve measurement data, and using the group of deviations and accelerations equations provided earlier, estimates of all twelve parameters at time K=2, can be generated . From this, the error covariance matrix at time k=2 can be computed, seeing that

$$P(2) = E\{[x(2) - \hat{x}(2)][x(2) - \hat{x}(2)]^T\}$$

From  $P(2)$ , the Kalman Predictor gain  $K(3)$  can be calculated, from which, estimates at time  $k=3$  can be generated. Such computational approach will lead to computation, in a sequential fashion, of estimates at times  $K=4, 5, 6, \dots$  etc.

An alternative initialization approach is to initialize the error covariance matrix by making it a diagonal matrix with variance values of 1. In essence, we overestimate the error covariance. Its only effect is to slow down the convergence of the filter. The filter itself builds up the covariance matrix, just as it builds up the gain.

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